Cryptography - *Day 2*

Implementations and Python

Review

Shift cipher

- $\mathcal{M}$ = {English word with lower case letters}
- Gen: choose uniform $k \in \mathcal{K} = \{0, \ldots, 25\}$
- $\mathrm{Enc}_k(m_1 \ldots m_t)$: output $c_1 \ldots c_t$, where

  $$c_i = [\, m_i + k \bmod 26]$$
- $\mathrm{Dec}_k(c_1 \ldots c_t)$: output $m_1 \ldots m_t$, where

  $$m_i = [\, c_i - k \bmod 26]$$
- Is this cipher secure? **No -- only 26 possible keys!**
  - Given a ciphertext, try decrypting with every possible key

Vigenere cipher

- $\mathcal{M}$ = {English word with lower case letters}
- Gen: choose uniform word $k=k_1 \ldots k_r \in \mathcal{M}$
- $\mathrm{Enc}_k(m_1 \ldots m_t)$: output $c_1 \ldots c_t$, where

  $$c_i = [m_i + k_j \bmod 26]$$
- $\mathrm{Dec}_k(c_1 \ldots c_t)$: output $m_1 \ldots m_t$, where

  $$m_i = [c_i - k_j \bmod 26]$$
- Is this cipher secure? **No – We can find the key length and the shift of each key!**

So far…

- "Heuristic" constructions; construct, break, repeat, …

- Can we *prove* that some encryption scheme is secure?

- First need to *define* what we mean by "secure" in the first place…

Core principles of modern crypto

- Formal definitions
  - Precise, mathematical model and definition of what security means
- Assumptions
  - Clearly stated and unambiguous
- Proofs of security
  - Move away from design-break-patch

Try Question 1

Quick Python!

First programming assignment

- Implement the Vigenère cipher. Then encrypt the message provided online.
- Will be posted after class.

Hexidecimal, ASCII, and XOR

Hexadecimal (base 16)

Hex	Bits (“nibble”)	Decimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7

Hex	Bits (“nibble”)	Decimal
8	1000	8
9	1001	9
A	1010	10
B	1011	11
C	1100	12
D	1101	13
E	1110	14
F	1111	15

Hexadecimal (base 16)

- 0x10
  - 0x10 = 16*1 + 0 = 16
  - 0x10 = 0001 0000

- 0xAF

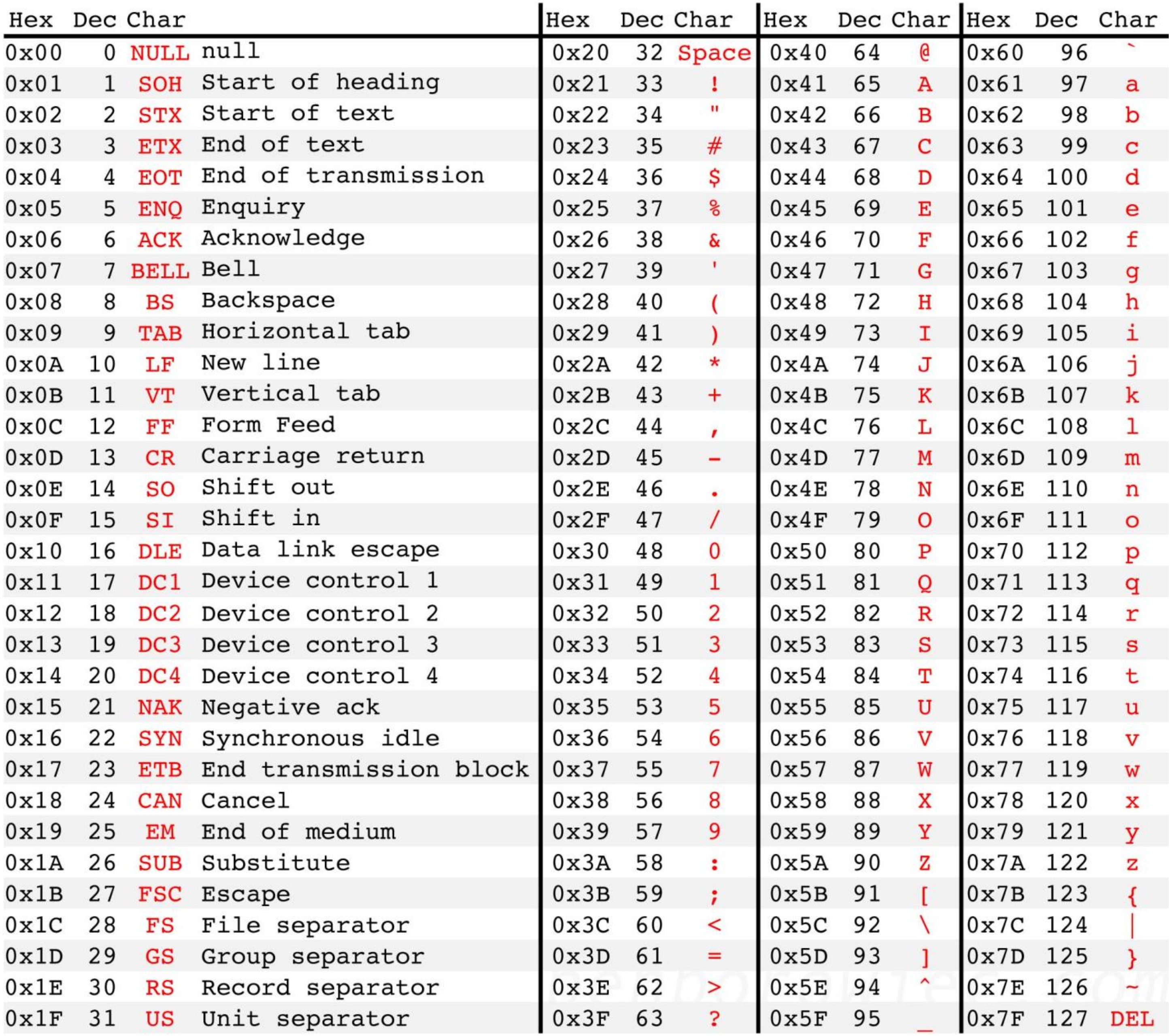
Hexadecimal (base 16)

- 0x10
  - 0x10 = 16*1 + 0 = 16
  - 0x10 = 0001 0000

- 0xAF
  - 0xAF = 16*A + F = 16*10 + 15 = 175
  - 0xAF = 1010 1111

ASCII

- Characters (often) represented in ASCII
  - 1 byte/char = 2 hex digits/char



Hex	Dec	Char		Hex	Dec	Char	Hex	Dec	Char	Hex	Dec	Char
0x00	0	NULL	null	0x20	32	Space	0x40	64	@	0x60	96	`
0x01	1	SOH	Start of heading	0x21	33	!	0x41	65	A	0x61	97	a
0x02	2	STX	Start of text	0x22	34	"	0x42	66	B	0x62	98	b
0x03	3	ETX	End of text	0x23	35	#	0x43	67	C	0x63	99	c
0x04	4	EOT	End of transmission	0x24	36	$	0x44	68	D	0x64	100	d
0x05	5	ENQ	Enquiry	0x25	37	%	0x45	69	E	0x65	101	e
0x06	6	ACK	Acknowledge	0x26	38	&	0x46	70	F	0x66	102	f
0x07	7	BELL	Bell	0x27	39	'	0x47	71	G	0x67	103	g
0x08	8	BS	Backspace	0x28	40	(	0x48	72	H	0x68	104	h
0x09	9	TAB	Horizontal tab	0x29	41	)	0x49	73	I	0x69	105	i
0x0A	10	LF	New line	0x2A	42	*	0x4A	74	J	0x6A	106	j
0x0B	11	VT	Vertical tab	0x2B	43	+	0x4B	75	K	0x6B	107	k
0x0C	12	FF	Form Feed	0x2C	44	,	0x4C	76	L	0x6C	108	l
0x0D	13	CR	Carriage return	0x2D	45	-	0x4D	77	M	0x6D	109	m
0x0E	14	SO	Shift out	0x2E	46	.	0x4E	78	N	0x6E	110	n
0x0F	15	SI	Shift in	0x2F	47	/	0x4F	79	O	0x6F	111	o
0x10	16	DLE	Data link escape	0x30	48	0	0x50	80	P	0x70	112	p
0x11	17	DC1	Device control 1	0x31	49	1	0x51	81	Q	0x71	113	q
0x12	18	DC2	Device control 2	0x32	50	2	0x52	82	R	0x72	114	r
0x13	19	DC3	Device control 3	0x33	51	3	0x53	83	S	0x73	115	s
0x14	20	DC4	Device control 4	0x34	52	4	0x54	84	T	0x74	116	t
0x15	21	NAK	Negative ack	0x35	53	5	0x55	85	U	0x75	117	u
0x16	22	SYN	Synchronous idle	0x36	54	6	0x56	86	V	0x76	118	v
0x17	23	ETB	End transmission block	0x37	55	7	0x57	87	W	0x77	119	w
0x18	24	CAN	Cancel	0x38	56	8	0x58	88	X	0x78	120	x
0x19	25	EM	End of medium	0x39	57	9	0x59	89	Y	0x79	121	y
0x1A	26	SUB	Substitute	0x3A	58	:	0x5A	90	Z	0x7A	122	z
0x1B	27	FSC	Escape	0x3B	59	;	0x5B	91	[	0x7B	123	{
0x1C	28	FS	File separator	0x3C	60	<	0x5C	92	\	0x7C	124	|
0x1D	29	GS	Group separator	0x3D	61	=	0x5D	93	]	0x7D	125	}
0x1E	30	RS	Record separator	0x3E	62	>	0x5E	94	^	0x7E	126	~
0x1F	31	US	Unit separator	0x3F	63	?	0x5F	95	_	0x7F	127	DEL

Source: http://benborowiec.com/2011/07/23/better-ascii-table/

ASCII

- ‘1’ = 0x31 = 0011 0001
- ‘F’ = 0x46 = 0100 0110

- Note that writing 0x00 to a file is different from writing “0x00” to a file
  - 0x00 = 0000 0000 (1 byte)
  - “0x00” = 0x30 78 30 30
    = 0011 0000 0111 1000… (4 bytes)

Day 2 - Worksheet

- Try Question 2 and Question 3 from the worksheet

Useful observations

- Only 128 valid ASCII chars (128 bytes invalid)
- 0x20-0x7E printable
- 0x41-0x7a includes upper/lowercase letters
  - Uppercase letters begin with 0x4 or 0x5
  - Lowercase letters begin with 0x6 or 0x7

XOR Operation

- XOR is a binary "exclusive or” operation that is represented by ⊕
- XOR is true if and only if the arguments differ
- Example: Evaluate the following.
  - 0100 1011 ⊕ 1010 0001
  - 0100 1000 ⊕ 0100 1000

Property of XOR

- **Lemma**. Suppose that b and b’ are binary numbers such that b = b’. Then b $\oplus$ b’ = e where e is the binary representation of zero.

Byte-wise shift cipher

- Work with an alphabet of *bytes* rather than (English, lowercase) *letters*
  - Works natively for arbitrary data!
- Use XOR instead of modular addition
  - Essential properties still hold

Byte-wise shift cipher

- $\mathcal{M}$ = {strings of bytes}
- Gen: choose uniform byte $k \in \mathcal{K} = \{0, \ldots, 255\}$
- $\text{Enc}_k(m_1 \ldots m_t)$: output $c_1 \ldots c_t$, where

  $$c_i := m_i \oplus k$$
- $\text{Dec}_k(c_1 \ldots c_t)$: output $m_1 \ldots m_t$, where

  $$m_i := c_i \oplus k$$

Example

- Say plaintext is “Hi” and key is

1010 0001 1111 0001

- “Hi” = 0x48 69 = 0100 1000 0110 1001

- XOR with “Hi” with the key

- 0100 1000 0110 1001 $\oplus$
  1010 0001 1111 0001
  = 1110 1001 1001 1000

Example

- Say plaintext is “Hi” and key is

1010 0001 1111 0001

- Ciphertext: 1110 1001 1001 1000 = 0xE9 98

Byte-wise Vigenère cipher

- The key is a string of bytes
- The plaintext is a string of bytes
- To encrypt, XOR each character in the plaintext with the next character of the key
  - Wrap around in the key as needed
- Decryption just reverses the process

Example

- Say plaintext is “Hello!” and key is 0xA1 2F
- “Hello!” = 0x48 65 6C 6C 6F 21
- XOR with 0xA1 2F A1 2F A1 2F
- 0x48 $\oplus$ 0xA1
  - 0100 1000 $\oplus$ 1010 0001 = 1110 1001 = 0xE9

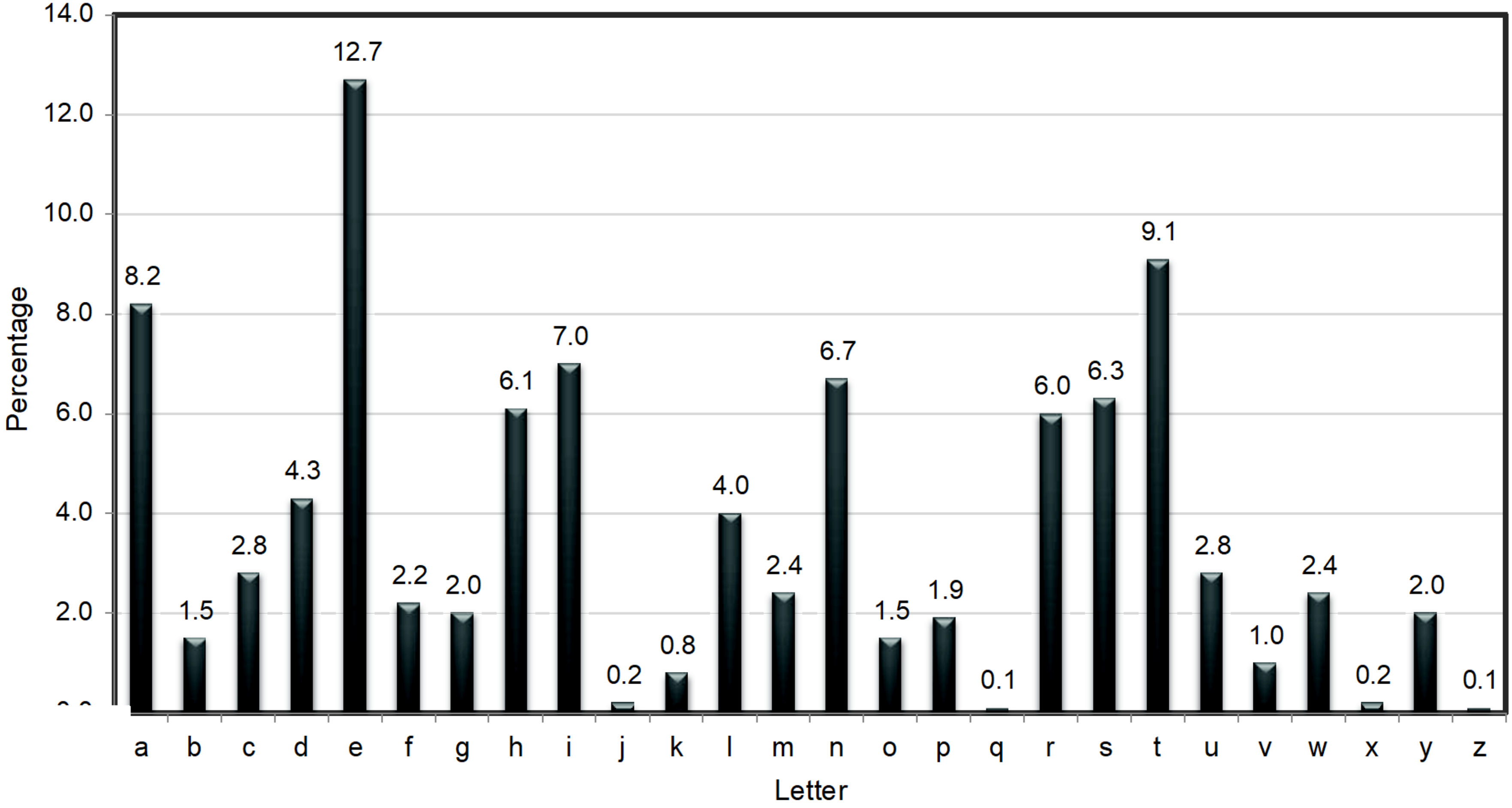
- Ciphertext: 0xE9 4A CD 43 CE 0E

Attacking the (variant) Vigenère cipher

- Two steps:
  - Determine the key length
  - Determine each byte of the key

- Same principles as before…

Using plaintext letter frequencies



Determining the key length

- Let p_i (for $0 \leq i \leq 255$) be the frequency of **byte** i in general English text
  - I.e., $p_i = 0$ for $i < 32$ or $i > 127$
  - I.e., p_{97} = frequency of ‘a’
  - The distribution is far from uniform

Determining the key length

- If the key length is N, then every Nth character of the plaintext is encrypted using the same “shift”
  - If we take every Nth character and calculate frequencies, we should get the p_i’s in permuted order
  - If we take every Mth character (M not a multiple of N) and calculate frequencies, we should get something close to uniform

Determining the key length

- How to distinguish these two?
- For some candidate key length, tabulate $q_0, \ldots, q_{255}$ and compute $\Sigma\, q_i^2$
  - If close to uniform, $\Sigma\, q_i^2 \approx 256 \cdot (1/256)^2 = 1/256$
  - If a permutation of p_i, then $\Sigma\, q_i^2 \approx \Sigma\, p_i^2$
    - Could compute $\Sigma\, p_i^2$ (but somewhat difficult)
    - Key point: will be much larger than 1/256
- Compute $\Sigma\, q_i^2$ for each possible key length, and look for maximum value
  - Correct key length should yield a large value for every stream

Determining the i^{th} byte of the key

- Assume the key length N is known
- Look at every N^{th} character of the ciphertext, starting with the i^{th} character
  - Call this the i^{th} ciphertext “stream”
  - Note that all bytes in this stream were generated by XORing plaintext with the same byte of the key
- Try decrypting the stream using every possible byte value B
  - Get a candidate plaintext stream for each value

Determining the i^{th} byte of the key

- Could use $\{p_i\}$ as before, but not easy to find
- When the guess B is correct:
  - All bytes in the plaintext stream will be between 32 and 127
  - Frequencies of lowercase letters (as a fraction of all lowercase letters) should be close to known English-letter frequencies
    - Tabulate observed letter frequencies $q'_0, \ldots, q'_{25}$ (as fraction of all lowercase letters)
    - Should find $\Sigma\, q'_i\, p'_i \approx \Sigma\, {p'_i}^2 \approx 0.065$, where p'_i corresponds to English-letter frequencies
    - In practice, take B that maximizes $\Sigma\, q'_i\, p'_i$