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**Directions**

1. Complete the following questions.
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1. The elements of  $\mathbb{Z}_{15}^*$  are  $\{1, 2, 4, 7, 8, 11, 13, 14\}$ . Define  $f_3(x) = [x^3 \bmod 15]$ . What is the result of applying  $f_3$  to the elements of  $\mathbb{Z}_{15}^*$ , in the order just presented?

(a)  $\{1, 2, 4, 7, 8, 11, 13, 14\}$

(b)  $\{1, 8, 4, 8, 4, 8, 2, 14\}$

(c)  $\{1, 8, 4, 13, 2, 11, 7, 14\}$

2. Which of the following is a generator of  $\mathbb{Z}_7^*$ ?

(a) 2

(b) 3

(c) 5

3. Let  $p, N$  be integers with  $p|N$ . Prove that for any integer  $X$ ,

$$[[X \bmod N] \bmod p] = [X \bmod p].$$

Show that, in contrast,  $[[X \bmod p] \bmod N]$  need not equal  $[X \bmod N]$ .